measurements for 0.625-mm- and 0.125-mm-diam upstream probes show that the wake effect is more pronounced in the former case and insigificant for the latter under similar plasma flow conditions mentioned previously.

These investigations indicate that the size and the orientation of a cylindrical probe may affect the measurements of the turbulent properties of a flowing plasma. The wake effect may be avoided using small diameter probes. However, since a very small diameter probe when used in a high temperature plasma jet gets red hot, the absolute electron density determination needs calibration procedure. In ballistic ranges, on the other hand, since the measuring time is short, a very small diameter probe can be used.

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Oscillations in Digital Control Systems

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Introduction

A PROBLEM in the implementation of digital controllers is the generation of low-amplitude oscillations by signal quantization. It has been shown[1] that if a digital control system is stable in a bounded-input bounded-output sense with no quantization, then the system will be stable in the same sense with quantization, provided that the controller operates with fixed-point arithmetic. However, it is well known that oscillations with bounded amplitudes may occur in such systems. An investigation of these oscillations is presented in this Note.

Development

The effects of quantization may be considered as a nonlinear gain, where this gain is a function of the signal being quantized. Consider a linear discrete system in which the effects of quantization are ignored.

$$\mathbf{x}(k+1) = A\mathbf{x}(k) + B\mathbf{u}(k) \tag{1}$$

With quantization present, this system may be modeled as

$$\mathbf{x}(k+1) = A_Q\mathbf{x}(k) + B_Q\mathbf{u}(k) \tag{2}$$

where, in general, A_Q and B_Q are nonlinear functions of the states and the inputs. Or,

$$A_Q = A_Q[\mathbf{x}(k),\ \mathbf{u}(k)] \tag{3}$$

$$B_Q = B_Q[\mathbf{x}(k),\ \mathbf{u}(k)] \tag{4}$$

A cause of low-amplitude oscillations in the system described by Eqs. (1) and (2) will now be investigated. Consider that the signal into one quantizer has settled into a certain quantization level for at least several consecutive sampling instants. Suppose that the system is modeled such that the signal out of this quantizer is $x_i(k)$. Then, for the several sampling instants,

$$x_i(k+1) = x_i(k) = x_i \tag{5}$$

The ith equation of Eq. (2) will be now Eq. (5). Thus, Eq. (2) can be written as

$$\mathbf{x}(k+1) = A_q\mathbf{x}(k) + B_q\mathbf{u}(k) + A_1\mathbf{x} = A_q\mathbf{x}(k) + [A_q,\ B_q]\begin{vmatrix}\mathbf{x}\\ \mathbf{u}(k)\end{vmatrix} \tag{6}$$

The matrices A_q and B_q can be obtained from A_Q and B_Q by considering the quantizer in question to be open. The matrix A_1 can be obtained by considering the output of this quantizer to be a source x_i, and ignoring the effects of $x_j(k)$, $j \neq i$, and $\mathbf{u}(k)$ on $\mathbf{x}(k+1)$, since these effects are included in A_q and B_q. Thus, in A_1 all columns are zero vectors except the ith column. The matrices are illustrated in the example below.

If A_q in Eq. (6) represents an unstable system, the input into the quantizer cannot remain within the quantization level for an indefinite period. For this case, with the systems inputs constant over a long period of time, the system states cannot settle to constant values, but instead will exhibit oscillations. These oscillations will be present whether fixed-point or floating-point arithmetic is used. The amplitude of the oscillations will, of course, depend on the digital wordlengths used within the controller.

Example

Consider the discrete model of the stable digital control system shown in Fig. 1. In this figure, the blocks that con-

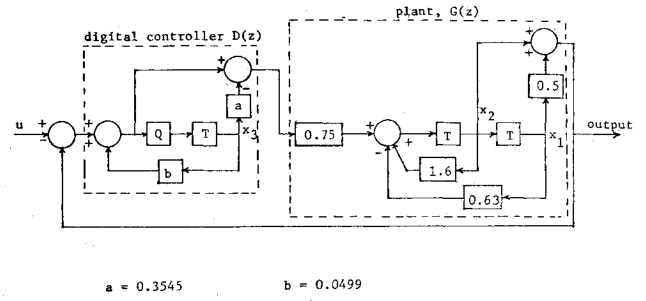

Fig. 1 Digital control system.

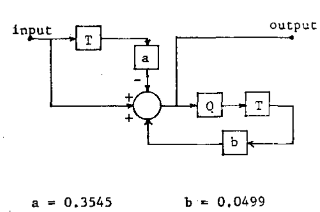

Fig. 2 Controller with direct programing form.

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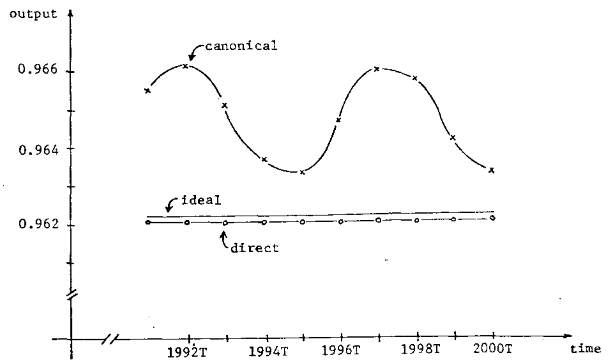

Fig. 3 Step responses for system.

tain T represent ideal delays of T sec. To simplify analysis, only one quantizer Q is considered. The open-loop system transfer function is

$$D(z)G(z) = \left[\frac{z - 0.3545}{z - 0.0499}\right]\left[\frac{0.75(z + 0.5)}{(z - 0.9)(z - 0.7)}\right] \quad (7)$$

where $D(z)$ is the transfer function of the controller, and $G(z)$ is the transfer function of the plant. The digital controller, when realized in the manner shown in Fig. 1, is said to be realized by the canonical programing form. The state equations for this system are

$$\mathbf{x}(k+1) = \begin{bmatrix} 0 & 1 & 0 \\ -1.005 & 0.85 & 0.2285 \\ -0.5Q & -Q & 0.3545Q \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 0 \\ 0.75 \\ Q \end{bmatrix} u(k) \quad (8)$$

In Eq. (8), Q is the nonlinear gain of the quantizer. In Fig. 1, if $x_3(k)$ becomes constant, Eq. (8) may be written as

$$\mathbf{x}(k+1) = \begin{bmatrix} 0 & 1 & 0 \\ -1.005 & 0.85 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 0 \\ 0.75 \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0.2285 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x} \quad (9)$$

Equation (9) represents an unstable system; thus the digital control system will contain oscillations.

These effects may also be viewed in terms of the poles and zeros of the controller. When $x_3(k)$ becomes constant over several sampling instants, no information concerning the changes that occur within the system is transmitted through the quantizer, and thus both the pole and the zero of the controller are in effect removed from the system. The system is unstable for this case.

(Consider the same system, with the controller realized by a different procedure. The diagram of this programing procedure is shown in Fig. 2, and is called the direct programing form.) For this case, if the quantizer transmits no information on changes within the system, only the pole of the controller is removed. The resultant system is stable, and no low-amplitude oscillations will be present.

Figure 3 illustrates the system unit-step response for the two programing forms, after approximately 2000 sampling instants. Floating-point arithmetic was used in the contoller, with three bits in the fraction. An expanded scale is used to clearly illustrate the oscillations. The ideal response of Fig. 3 is the system response with no quantization present. These results were obtained from a digital simulation.

Conclusions

A cause of low-amplitude oscillations in digital control systems is presented. It is illustrated through an example that the oscillations are related to the programing form used in the digital controller. This relationship can be explained from the effect of the quantizer on the poles and zeros of the digital controller, when the quantizer is transmitting no information.

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Viscous Slipstream Flow Downstream of a Centerline Mach Reflection

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Nomenclature

b = shear layer half-width
l = shock stem height, 0.34 in.
M = Mach number
p = static pressure
p_t' = Pitot pressure
r = radial distance
R = gas constant
z = axial distance
T = temperature
u = axial velocity
v' = transverse turbulent velocity, $(\overline{v'^2})^{1/2}$
γ = specific heat ratio
μ = viscosity
ρ = density
ψ = stream function

Subscripts

$\mathcal{C}$ = centerline condition
0 = reservoir condition
s = condition at edge of slipstream
t = stagnation condition
1 = condition upstream of shock stem
2 = condition just downstream of shock stem

Introduction

An important aspect of supersonic flows with shock waves is the reflections of these waves at boundaries such as along centerlines, along surfaces, and along free jet extremities. The first of these interactions is considered herein for a situation where the reflection from the centerline in an axisymmetric flow is through a shock stem (Mach reflection). The wave pattern is depicted in Fig. 1. The incident shock wave is S_i, the reflected shock wave is S_r, and the stem shock wave is S_n; a slipstream emanates from the triple shock wave intersection T. The purpose of this investigation is to determine the mean structure of the viscous flowfield downstream of the intersection from Pitot and static pressure probe measurements. There is virtually no experimental information available on the structure of such a shear flow and on the size of the subsonic flow region that is imbedded in the supersonic flow.

This region is of interest because such a shock wave reflection is often found in supersonic flows. Examples are 1) inside of channels where shock waves generated upstream in the flow by compressive turning subsequently undergo reflections from the centerline and the boundary layer along the

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